Caringbah High School

2015

Trial HSC Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using a blue or black pen.
  Black pen is preferred.
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of the paper
- In Questions 11 – 16, show all relevant mathematical reasoning and/or calculations.

Total Marks – 100

Section I – 10 marks

- Attempt Questions 1 – 10
- Allow approximately 15 minutes for this section

Section II – 90 marks

- Attempt Questions 11 – 16
- Allow approximately 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

For Questions 1–10, use the multiple-choice answer sheet on page 18. Please detach this from the exam paper and submit with your answer booklets.

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1 Evaluate $\log_e(\sin 2.5)$, correct to 3 significant figures. **1**

(A) -0·223

(B) -0·513

(C) -1·36

(D) -3·13

2 $(x+2)$ is a factor of which expression. **1**

(A) $x^3 - 8$

(B) $x^3 + 8$

(C) $x^2 + 4$

(D) $x^2 - 4x + 4$

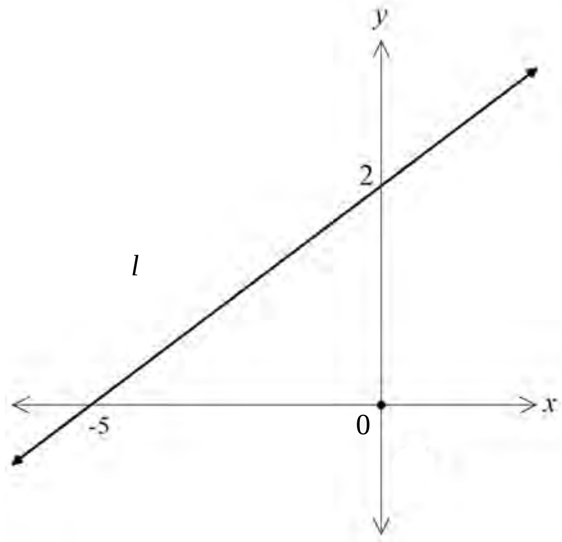
3 The diagram shows the line, *l*. Find the gradient of the line, *l*. **1**

(A) $\frac{-5}{2}$

(B) $\frac{-2}{5}$

(C) $\frac{2}{5}$

(D) $\frac{5}{2}$



4 What is the derivate of $y = \dfrac{e^x}{e^x + 1}$ **1**

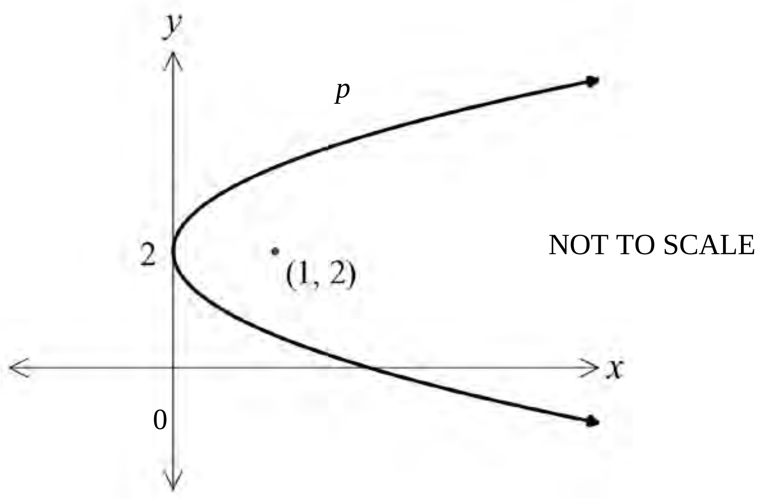
(A) $\ln\left(e^x + 1\right)$

(B) $\ln\left(e^x + 1\right)^2$

(C) $\dfrac{2e^{2x} + e^x}{\left(e^x + 1\right)^2}$

(D) $\dfrac{e^x}{\left(e^x + 1\right)^2}$

5 This graph shows the parabola, *p*, with vertex at 2 on the *y*-axis and focus $(1,\ 2)$ **1**



Which equation represents the parabola, *p*?

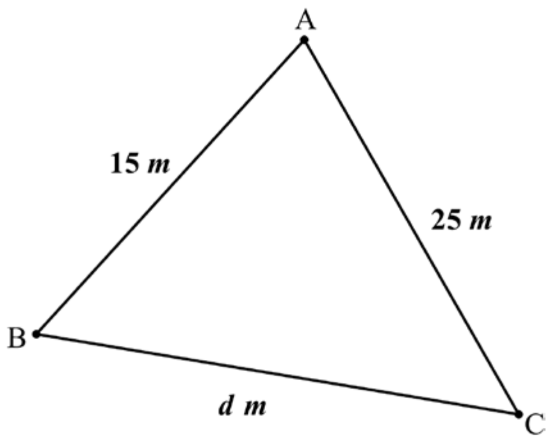
(A) $y^2 = 8(x - 2)$

(B) $(y - 2)^2 = 8x$

(C) $y^2 = 4(x - 2)$

(D) $(y - 2)^2 = 4x$

6 In ΔABC $AB = 15\ m$, $AC = 25\ m$ and $\angle ACB = 40^\circ$. **1**



NOT TO SCALE

Which expression can be used to find *d*?

(A) $d^2 = 15^2 + 25^2 - 2 \times 15 \times 25 \cos 40°$

(B) $d^2 = 15^2 + 25^2 - 2 \times 15 \times 25 \sin 40°$

(C) $15^2 = 25^2 + d^2 - 2 \times d \times 25 \cos 40^\circ$

(D) $15^2 = 25^2 + d^2 - 2 \times d \times 25 \sin 40°$

7 Find the primitive of $4\sin(2x+3)$ **1**

(A) $-2\cos(2x+3)$

(B) $-8\cos(2x+3)$

(C) $2\cos(2x+3)$

(D) $8\cos(2x+3)$

8 Which inequality defines the domain of the function $f(x)=\dfrac{x^2-9}{\sqrt{4-x^2}}$ **1**

(A) $-2<x<2$

(B) $-3<x<3$

(C) $-2\le x\le 2$

(D) $-3\le x\le 3$

9 Which of the following describes the sequence **1**

$$\ln(x)+\ln(x^2)+\ln(x^3)+\ln(x^4)+\ldots\ldots\ldots\ldots$$

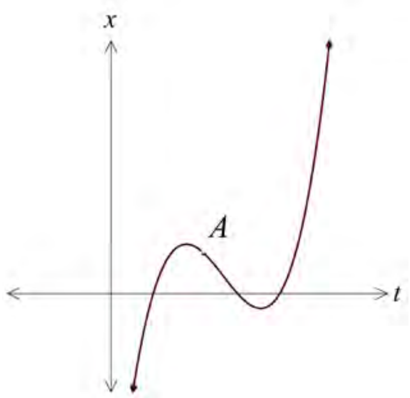
(A) Arithmetic sequence

(B) Geometric sequence with a limiting sum

(C) Geometric sequence without a limiting sum

(D) Neither an arithmetic nor a geometric sequence

10 The diagram shows the displacement, x metres, of a moving object at time t seconds. **1**



Which of the following statements describes the motion of the object at the point A

(A) Velocity is negative and acceleration is positive.

(B) Velocity is negative and acceleration is negative.

(C) Velocity is positive and acceleration is negative.

(D) Velocity is positive and acceleration is positive

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet.

(a) Given that $x^5 = 10\ 000$, find x correct to 3 significant figures. **2**

(b) Evaluate $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 9}$. **2**

(c) Sketch the graph of $y = x^3 - 8$, showing intercepts with the coordinate axes. **2**

(d) Differentiate $\log_e \left(\cos^2 x\right)$, expressing your answer in simplest form. **2**

(e) Differentiate $x^2 \sin x$. **2**

(f) The derivative function of a curve $y = f(x)$ is given by $f'(x) = 6x^2 - 2$. **2**
The curve passes through the point $(1, 1)$.

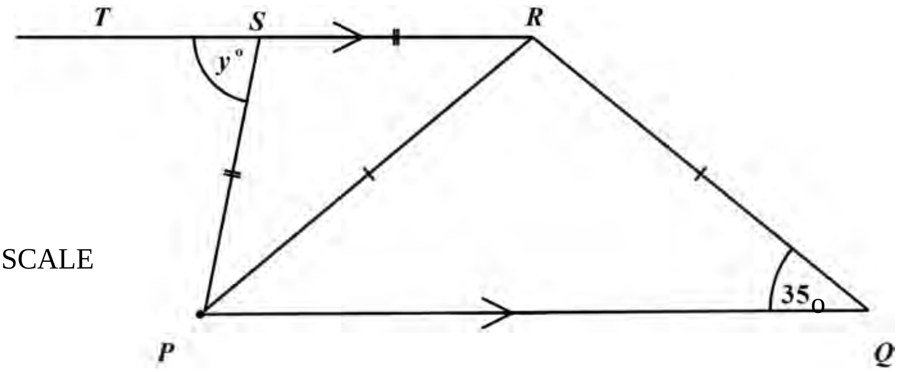
Find the equation of the curve.

Question 11 continues on page 7

Question 11 (continued)

(g) The diagram shows a quadrilateral $PQRS$, in which $PQ \parallel SR$, $PS = SR$ and $PR = RQ$. T is a point on RS produced.

NOT TO SCALE



Copy or trace this diagram into your writing booklet.

(i) Given that $\angle RQP = 35^\circ$, explain why $\angle PRQ = 110^\circ$ **1**

(ii) If $\angle TSP = y^\circ$ find a value for y giving reasons. **2**

End of Question 11

Question 12 (15 marks) Use the Question 12 Writing Booklet.

(a) $\int (2x-5)^3 \, dx$ **2**

(b) Evaluate $\int_{\frac{\pi}{12}}^{\frac{\pi}{9}} \cos 3x \, dx$ leaving your answer in exact form. **3**

(c) Evaluate $\sum_{k=15}^{36} (3k-2)$. **2**

(d) The vertices of ΔAOB are the points $A(0,\ 4)$, $O(0,\ 0)$ and $B(6,\ -2)$.

(i) Draw a diagram of ΔAOB ($\frac{1}{3}$ page), labelling all points clearly. Also **1**
mark in the point *K* that lies on the interval *AB* such that $OK \perp AB$.
(You do not need to find the coordinates of *K*).

(ii) By finding the gradient of *AB*, show that the equation of the line *AB* is given by, $x + y - 4 = 0$. **2**

(iii) By finding the distance *OK*, show that the area of ΔAOB is $12u^2$. **2**

(iv) A horizontal line through *B* meets *KO* produced at *S*. Find the coordinates of *S*. **1**

(v) Verify that $AS \perp BO$. **2**

End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet.

(a) Sydney is 694 nautical miles (M) south of Noumea and 761 M west. Calculate the distance (to the nearest nautical mile) and bearing (to the nearest degree) a plane must fly to make a trip from Noumea to Sydney. **3**

(b) Sketch the graph of $y = 3\sin 2x$ for $0 \le x \le \pi$. **2**

(c) If $\sin x = \frac{-2}{5}$ and $\cos x > 0$, find the exact value of $\cot x$. **1**

(d) In this diagram *ABCD* and *DEFG* are both squares. **3**

Prove $AE = CG$.

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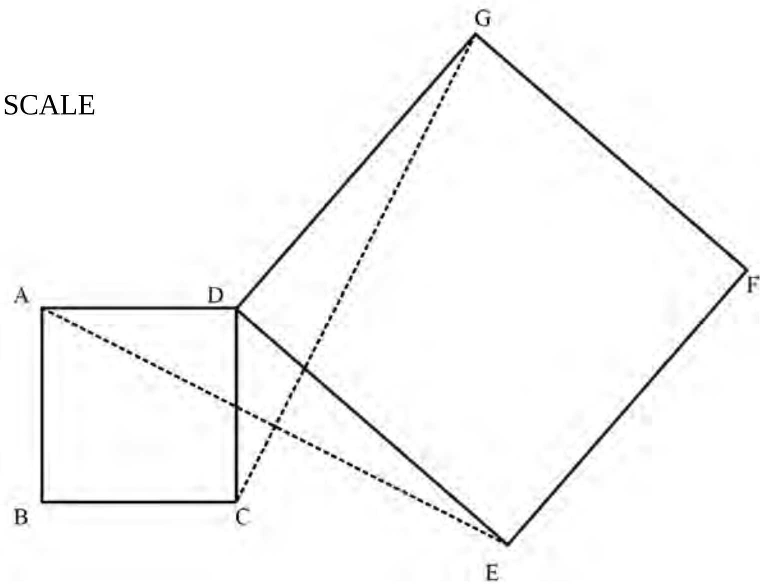
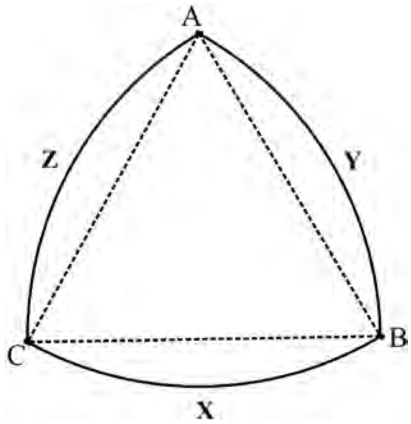


(e) Find the equation of the normal to the curve $y = \ln(3x+4)$ at the point $x = -1$ **3**

Question 13 continues on page 10

Question 13 (continued)

(f) A cam in a lock, AYBXCZ, is drawn below.



NOT TO SCALE

ABC is an equilateral triangle of side length 12cm. The arcs *AYB*, *BXC* and *CZA* have centres *C*, *A* and *B* respectively.

(i) Find the perimeter of the cam. **1**

(ii) Find the exact area of the cam. **2**

End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet.

(a) Find the values of x for which $|2x-1| \leq 3$. **2**

(b) Solve $2\sin\left(x-\frac{\pi}{4}\right)+1=0$ for $0 \leq x \leq 2\pi$. **2**

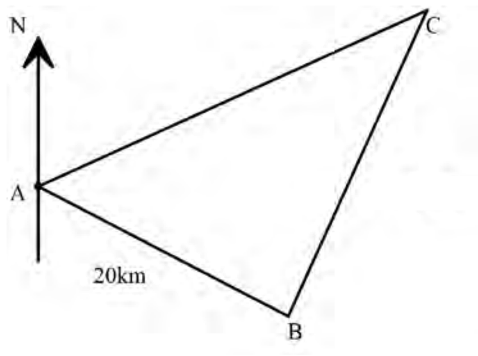
Leaving your answer(s) in exact form.

(c) Find the values of m for which the graph of the parabola $y=4x^2-mx+9$

(i) touches the x-axis. **1**

(ii) crosses the x-axis. **1**

(d) An ocean sailing regatta has yachts sailing in a triangular course as shown. **3**



NOT TO SCALE

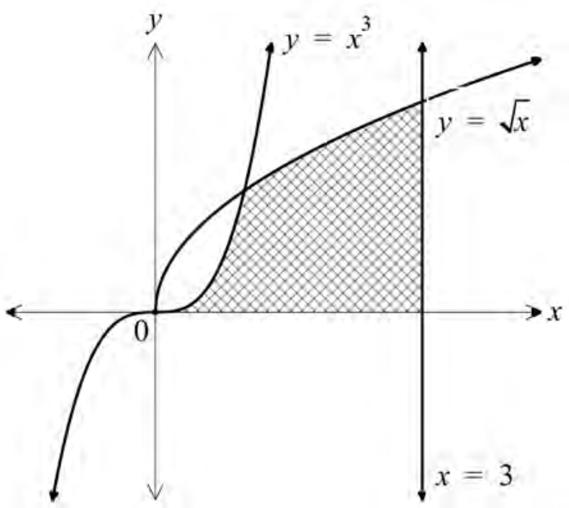
They sail from A for 20 km on a bearing of 150° T to the turning mark a B. The second leg from B to C is on a bearing of 020° T.

By determining the size of angle ABC, find the bearing of the third leg from C to A if it is known that C is 40 km from A.

Question 14 continues on page 12

Quesstion 14 (continued)

(e) The area bounded by the curves $y = x^3$, $y = \sqrt{x}$, the x-axis and the line $x = 3$ is shown. **3**



NOT TO SCALE

Find the area of the enclosed region.

(f) The area bounded by the curve $y = \sec x$ and the x-axis for $0 \le x \le \frac{\pi}{3}$ is rotated about the x-axis. **3**

Find the volume of the solid of revolution formed by this rotation.

End of Question 14

Question 15 (15 marks) Use the Question 15 Writing Booklet.

(a) Solve these simultaneous equations. **2**

$$4x - y = 3$$

$$10x + 3y = 2$$

(b) A point $P(x, y)$, moves so that its distance from the line $y = -2$ is equal to its distance from the point $S(5, 2)$. **2**

Find the equation of the locus of *P*.

(c) An object moves so that its velocity, *v m/s* , at any time, *t* seconds, is given by,

$$v = e^{-2t}$$

(i) Show that the acceleration is always negative. **1**

(ii) Find the acceleration after 1 second. **1**

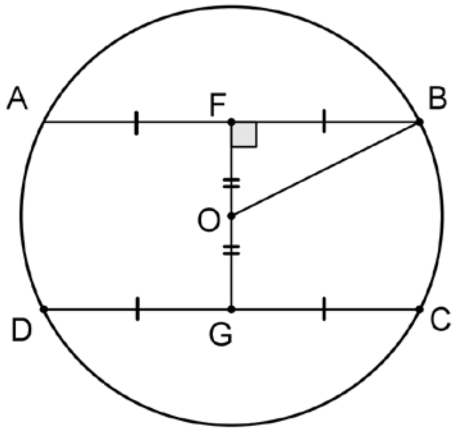
(iii) If the object is initially 2 m to the right of the origin, find an expression for the displacement of *x* in terms of *t*. **2**

(iv) Describe the motion of the particle as time increases. Include a description of displacement, velocity and acceleration. **2**

Question 15 continues on page 14

Question 15 (continued)

(d) A circular stained glass window of radius 3 m requires metal strips for support along *AB*, *DC* and *FG*, as shown in the diagram.



Copy the diagram and information into your writing booklet.

O is the centre of the circle.
Let $OF = OG = y$ metres and $FB = FA = GC = GD = x$ metres.

(i) Find an expression for *y* in terms of *x*. **1**

(ii) The total length of the support strips (ie. *AB* + *DC* + *FG*) is *L* metres. **1**

Show $L = 4x + 2\sqrt{9 - x^2}$

(iii) The window will have a maximum strength when the length of its supports is a maximum. **3**

Show that $FB = \frac{6\sqrt{5}}{5}$ metres, provides maximum strength for this window.

End of Question 15

Question 16 (15 marks) Use the Question 16 Writing Booklet.

(a) In January 2010 Jane decided to start saving for a holiday when she finishes Year 12.

She put $10 into a shoe box for safe keeping. The next month, she added to her savings by placing $12 in the shoe box. Each month she put in $2 more than the previous month. In October 2015, Jane will place her final amount into the box.

(i) How much will Jane put in the shoe box in October 2015? **2**

(ii) Calculate the total amount of her savings. **1**
(Note: Jane's shoe box does not offer any interest).

(b) Radioactive substances decay over time. It is known that a mass, *M*, of a substance that remains after *t* years satisfies the equation,

$$M = M_{\circ}e^{-kt}$$

Where $M_{\circ}$ and k are constant.

The radioactive isotope radium 226 has a half-life of approximately 1600 years.
(Note: The half-life of a substance is the time taken for half the material to decay).

(i) Find a value for *k*, expressing your answer in scientific notation correct to 3 significant figures. **1**

(ii) If ANSTO has a piece of radium 226 that weighs 200g, how much of this piece of radium existed in the year 1000 BCE (3015 years ago)? **2**

Question 16 continues on page 16

Question 16 (continued)

(c) The distance travelled by an object can be calculated by finding the area bounded by the graph of the velocity and the horizontal axis for time. **3**

An object moves in a way that satisfies the equation,

$$v = 4 - \sin^2 t$$

Use Simpson's rule with 3 function values to find an approximate distance travelled by the object in the first 4 seconds.

(d) At the completion of his degree, Oliver had a HEC's debt of \$100 000. He plans to repay this in equal monthly repayments of \$*M*. Interest is charged at a rate of 0.5% per month.

(i) Show that the amount owing after 3 months, A_3, is given by **1**

$$A_3 = 100\,000 \times 1 \cdot 005^3 - M\left(1 + 1 \cdot 005 + 1 \cdot 005^2\right)$$

(ii) If Oliver wishes to pay off his loan by the end of 10 years, then show that he will need to pay \$1 110 per month. **2**

(iii) Show that the amount owing after *n* months can be written as, **1**

$$A_n = 1 \cdot 005^n \left[100\,000 - 200M\right] + 200M$$

(iv) If Oliver decides that he can only repay \$750 each month, how long will it take him to repay the loan? **2**
(Answer in years and months)

End of paper

CHS TRIAL 2U 2015

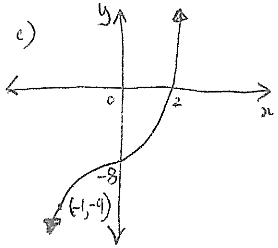
MC. B, B, C, D, D, C, A, A, A, B.

Q11

a) 6.31 (3 sig fig)

b) $\lim_{x\to 3} \frac{(x-3)(x+2)}{(x-3)(x+3)} = \frac{5}{6}$

c)



d) $y = \ln(\cos^2 x)$

$y' = \frac{-2\cos x \sin x}{\cos^2 x}$

$= -2\tan x$

e) $y = x^2 \sin x$

$y' = x^2\cos x + 2x\sin x$

f) $y' = 6x^2 - 2$

$y = 2x^3 - 2x + c$

$x=1, y=1 \therefore c = 1$

$y = 2x^3 - 2x + 1$

g) i) $\angle RPQ = 35^\circ$ angles opp. eq sides of isos. Δ

$\angle PRQ = 110$ $(180-(35+35))$ angle sum ΔPRQ

ii) $\angle SRQ = 145$ $(180-35)$ coint $\angle$s, $SR \parallel PQ$.

$\therefore \angle SRP = 35^\circ$ $(145-110)$

$\angle SPR = 35^\circ$ angles opp eq. sides of isos Δ.

$\therefore \angle TSP = 70$ ext $\angle$ of ΔSRP.

Q12

a) $\int (2x-5)^3\,dx = \frac{(2x-5)^4}{8} + c$

b) $\frac{1}{3}\left[\sin 3x\right]_{\frac{\pi}{12}}^{\frac{\pi}{9}} = \frac{1}{3}\left[\sin\frac{\pi}{3} - \sin\frac{\pi}{4}\right]$

$= \frac{1}{3}\left(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right)$

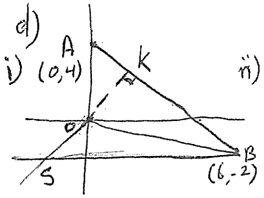
$= \frac{\sqrt{3}-\sqrt{2}}{6}$

c) $43 + 46 + 49 + \ldots + 106$ $a = 43$, $d = 3$

$S_{22} = \frac{22}{2}[43 + 106]$

$= 1639$

d)



i) $(0,4)$

ii) $M_{AB} = \frac{4+2}{0-6}$

$= -1$

$(y-4) = -1(x-0)$

$y = 4 - x$

$x + y - 4 = 0$

iii) $OK = \left|\frac{0+0-4}{\sqrt{1+1}}\right|$

$= \frac{4}{\sqrt{2}}$

$A = \frac{1}{2} \times \frac{4}{\sqrt{2}} \times \sqrt{(4+2)^2 + (0-6)^2}$

$= \frac{1}{2} \times \frac{4}{\sqrt{2}} \times 6\sqrt{2}$

$= 12u^2$

iv) Let S be (x, y)

$M_{OS} = -1$ $\frac{y-0}{x-0} = -1$ $y = -x$

but $y = -2$ so $x = -2$

$S(-2, -2)$

v) $m_{AS} = \frac{6}{2}$ $m_{BO} = \frac{-2}{6}$

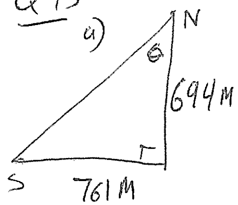
$m_{AS} \times m_{BO} = \frac{6}{2} \times -\frac{2}{6}$

$= -1$

$\therefore AS \perp BO$

Q13

a)



$d = \sqrt{761^2 + 694^2}$

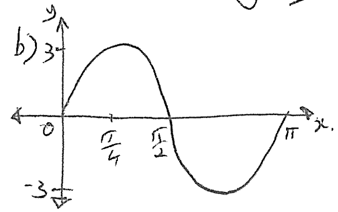
$= 1030$ M

$\tan\theta = \frac{761}{694}$

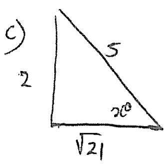
$\theta = 48°$

bearing $= 228^\circ$

b)



c)



x is in quad 4 $\therefore \cot x < 0$

$\cot x = -\frac{\sqrt{21}}{2}$

d) In ΔADE and ΔCDE

$AD = CD$ and $DE = EG$ sides of squares

If $\angle CDE = x^\circ$ $\angle ADE = \angle CDG = 90 + x$. ($90^\circ$ in squares)

$\therefore \Delta ADE \equiv \Delta CDE$ SAS

$\therefore AE = CG$ corr. sides of cong Δs

e) $y' = \frac{3}{3x+4}$ $x = -1$ $m_T = 3$, $m_N = -\frac{1}{3}$

$x = -1$, $y = \ln 1 = 0$

Eqn $y - 0 = -\frac{1}{3}(x+1)$

$x + 3y + 1 = 0$

f) i) $l = r\theta$ $P = 3l$

$= 12 \times \frac{\pi}{3}$ $= 12\pi$

ii) $A = \frac{1}{2}r^2\theta + 2\left(\frac{1}{2}r^2(\theta - \sin\theta)\right)$

$= \frac{1}{2} \times 12^2 \times \frac{\pi}{3} + 2\left(\frac{1}{2} \times 12^2\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)\right)$

$= 24\pi + \frac{144\pi}{3} - 144 \times \frac{\sqrt{3}}{2}$

$= 72\pi - 72\sqrt{3}$

Q14

a) $2x - 1 \le 3$ $2x - 1 \ge -3$

$x \le 2$ $x \ge -1$

$-1 \le x \le 2$

b) $\sin\left(x - \frac{\pi}{4}\right) = -\frac{1}{2}$

$x - \frac{\pi}{4} = \frac{7\pi}{6}$ $x - \frac{\pi}{4} = \frac{11\pi}{6}$

$x = \frac{17\pi}{12}$ $x = \frac{25\pi}{12} = \frac{\pi}{12}$

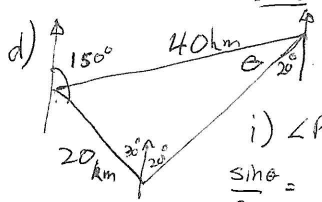
c) $b^2 - 4ac = m^2 - 4 \times 36$

i) $\Delta = 0$ $m^2 = 4 \times 36$

$m = \pm 12$

ii) $\Delta > 0$ $m < -12$ or $m > 12$

d)



i) $\angle ABC = 50$

$\frac{\sin\theta}{20} = \frac{\sin 50}{40}$

$\theta = 23^\circ$

$\therefore$ bearing $= 223^\circ$

e) $x^3 = \sqrt{x}$ $x = 0$ and $x = 1$

$A = \int_0^1 x^3\,dx + \int_1^3 x^{\frac{1}{2}}\,dx$

$= \frac{1}{4}\left[x^4\right]_0^1 + \frac{2}{3}\left[x^{\frac{3}{2}}\right]_1^3$

$= \frac{1}{4} + \frac{2}{3}(\sqrt{27} - 1)$

$= \frac{8\sqrt{27} - 5}{12}$

f) $V = \pi\int_0^{\frac{\pi}{3}} \sec^2 x\,dx$

$= \pi\left[\tan x\right]_0^{\frac{\pi}{3}}$

$= \pi\left[\tan\frac{\pi}{3} - \tan 0\right]$

$= \sqrt{3}\pi$

Q15 a) $4x - y = 3$ ①

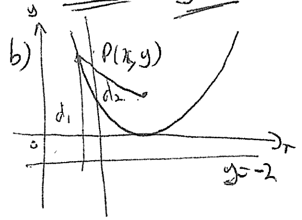
$10x + 3y = 2$ ②

①×3 $12x - 3y = 9$ ③

②+③ $22x = 11$

$x = \frac{1}{2}$ $y = -1$

b)



Parabola $V(5,0)$ $a=2$.

$(x-5)^2 = 8y$

or

$d_1^2 = d_2^2$ $(y+2)^2 = (x-5)^2 + (y-2)^2$

$(x-5)^2 = (y^2+4y+4) - (y^2-4y+4)$

$= 8y$

c) i) $v = e^{-2t}$

$\frac{dv}{dt} = -2e^{-2t}$ as $e^{-2t} > 0$

$\frac{dv}{dt} < 0.$

ii) $t = 1$

$a = -2e^{-2}$

iii) $x = -\frac{1}{2}e^{-2t} + c$

$t=0$ $x=2$ ∴ $c = 2\frac{1}{2}$

$x = -\frac{1}{2}e^{-2t} + 2\frac{1}{2}.$

iv) As $t \to \infty$ $e^{-2t} \to 0$

∴ $x \to 2\frac{1}{2}$

$v \to 0$, $a \to 0$.

ie slowing down approaching $2\frac{1}{2}$.

d) i) $x^2 + y^2 = 9$ ∴ $y = \sqrt{9-x^2}$

ii) $L = 4x + 2y = 4x + 2\sqrt{9-x^2}$

iii) $L' = 4 + (9-x^2)^{-\frac{1}{2}} \times -2x.$

max $L' = 0$ $\frac{2x}{\sqrt{9-x^2}} = 4$

$\frac{4x^2}{9-x^2} = 16$

$4x^2 = 144 - 16x^2$

$20x^2 = 144$

$x = \frac{12}{\sqrt{20}}$ (as $x > 0$)

$= \frac{6\sqrt{5}}{5}$

check for max

x	1	$\frac{6\sqrt{5}}{5}$	2·9
L'	<0		>0

∴ max at $x = \frac{6\sqrt{5}}{5}$

Q16

a) $\$10 + \$12 + \$14 + \ldots$

$a = 10$ $d = 2$ $n = 5 \times 12 + 10$

$= 70$

i) $T_n = 10 + 69 \times 2$

$= \$148$

ii) $S_n = \frac{70}{2}(10 + 148)$

$= \$5530$

b) When $t = 1600$ $M = \frac{1}{2}M_0$

i) $\frac{1}{2}M_0 = M_0 e^{-1600k}$

$-1600k = \ln\frac{1}{2}$

$k = -\frac{\ln\frac{1}{2}}{1600}$ $\left(\frac{\ln 2}{1600}\right)$

$= 4.33 \times 10^{-4}$

ii) $200 = M_0 e^{-3015k}$

$M_0 = 200 \div e^{-3015k}$

$= 738g$

c) $d = \int_0^4 4 - \sin^2 t$

0	2	4	$h=2$
4	$4-\sin^2 2$	$4-\sin^2 4$	

$d \doteq \frac{2}{3}[4 + (4-\sin^2 4) + 4(4-\sin^2 2)]$

$= 13.4m$

Q16 d

i) $A_1 = 100\,000 \times 1.005 - M$

$A_2 = 100\,000 \times 1.005^2 - M \times 1.005 - M$

$A_3 = 100\,000 \times 1.005^3 - M \times 1.005^2 - M \times 1.005 - M$

$= 100\,000 \times 1.005^3 - M(1 + 1.005 + 1.005^2)$

ii) 10 yrs ⇒ $n = 120$

$A_n = 100\,000 \times 1.005^n - M(1 + 1.005 + \ldots + 1.005^{n-1})$

$M = \frac{100\,000 \times 1.005^{120}}{\left[\frac{1.005^{120}-1}{0.005}\right]}$

$= \$1\,110.20$

iii) $A_n = 100\,000 \times 1.005^n - M\left[\frac{1.005^n - 1}{0.005}\right]$

$= 100\,000 \times 1.005^n - 200M[1.005^n - 1]$

$= 1.005^n[100\,000 - 200M] + 200M.$

iv) $1.005^n[100\,000 - 200 \times \$750] + 200 \times \$750 = 0$

$1.005^n = \frac{-200 \times 750}{100\,000 - 200 \times 750}$

$= \frac{150\,000}{50\,000}$

$n = \frac{\ln 3}{\ln 1.005}$

$= 220$ mths

$= 18$ yrs 5 mths